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Application of the Dimensionless Load-Load Point Displacement Curve to Analyze the Non-Linear Fracture Parameters of Dental Resin

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Abstract *To establish a method for determining the non-linear fracture parameters (crack growth resistance Rc, non-linear energy fracture toughness Gc, plastic energy dissipation rate ϕ_{nl}) of dental resin, the dimensionless load-load point displacement ($\tilde{P}$-$\tilde{u}$) curve was applied to heat cured denture base resin. The parameters were given by the $\tilde{P}$-$\tilde{u}$ curve of both an ideal elastic body and a pre-cracked specimen obtained from the load-displacement curve during stable crack growth, the fracture toughness and the bending modulus. The analysis revealed that the non-linear fracture parameters could be determined quantitatively without an unloading-reloading procedure, and that the ϕ_{nl} resulting from the plastic deformation was greater than the net energy Gc required for the formation of the crack surface, and the ϕ_{nl} held the greater part of the Rc equivalent to the total energy necessary for stable crack growth.*

Key words : Dental resin, Non-linear fracture parameter, Dimensionless load-load point displacement curve

INTRODUCTION

More than forty years ago, Smith[1] reported that maxillary complete dentures suffered midline fracture after about 15 to 30 months in clinical use, and pointed out that the cracking was due to fatigue failures. Even now, the cause of the cracking in denture bases is still an open question[2,3]. The mechanical fatigue of a material is an important factor in crack growth[4]. When the crack is extending in a material, the breakdown of bond follows the stretch of bond distance between atoms or molecules over the limit of elastic range. Consequently, as crack growth shows the plastic deformation near the crack tip, it should be analyzed by non-linear fracture parameters.

Sakai and Bradt[5] introduced the concept of dimensionless load ($\tilde{P}$) and dimensionless load point displacement ($\tilde{u}$) to determine the nonlinear fracture energy of ceramics such as refractory composites during crack extension. The $\tilde{P}$-$\tilde{u}$ curve is obtained by converting the measured load (P) and load point displacement (u) using the fracture toughness (K_{IC})[6] and the bending modulus (E). An impressive feature of this method is that the $\tilde{P}$-$\tilde{u}$ curve for an arbitrary ideal elastic body gives universal relation not only in ceramics but also metals and resins. In the present study, this method was applied to the heat cured denture base resin, in order to evaluate whether the nonlinear fracture parameters of dental resin could be determined or not. The bending modulus (E) was measured to convert the P-u curve into the $\tilde{P}$-$\tilde{u}$ curve.

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MATERIALS AND METHODS

1. Preparation of specimen

The specimen preparation followed the previous study[7,8]. Namely, the heat cured denture base resin (Acron clear No. 5, GC Co., Tokyo, Japan) was used as a dental resin. After polymerization at a powder-liquid ratio of 100g/34.1g in the shape of plate, beam specimens (4 (B)×8 (W)×50 mm) were cut off from the plate according to ASTM D5045-957). Some of them were used to measure the bending modulus (E) after being kept in distilled water for more than 1 week. The remainder were notched by the diamond notch blade and dried at 50℃, and then a pre-crack (4.0±0.1 mm) was produced by the three-point bend mode[6]. After that, these were also kept in distilled water before measurement.

2. Measurement of bending modulus (E)

The bending modulus (E) was measured by the three point bend method with a support span (S) of 32 mm, using a material testing machine (Autograph AG-5000C, Shimadzu Co., Kyoto, Japan). The bending jig was placed in a glass vessel and the specimen was covered with the distilled water. Both the time and the load (P) in measuring at a cross head speed of 0.001 mm/min were recorded, and the load point displacement (u) was calculated from the time and the cross head speed. To make corrections in u arising from the elastic strain of the testing system itself, such as the jig, load cell and testing machine, the P_m-u_m curve of a thick metal beam which could be regarded as a rigid material compared with the resin was measured. The u_m was considered as elastic displacement of the testing system, and deducted from that of the resin specimen. E was calculated from the corrected P-u curve using Equation (1),

$$E=(S^3/4W\cdot B^3)((P_2-P_1)/(u_2-u_1)) \quad (1)$$

where P_1 and P_2 are loads at the displacement of u_1 and u_2, respectively, and W (=8 mm) and B (=4 mm) are the width and the thickness, respectively.

3. Load-load point displacement (P-u) curve during stable crack growth (SCG)

As illustrated in Fig. 1, the P-u curve of the pre-cracked specimen during a stable crack growth (SCG) was measured by the same device and method used for the E measurement. Similar correction for the elastic strain of the testing system was also made in the P-u curve. Photographs of the extending crack were taken from an oblique direction toward the crack surface with an optical digital microscope (VH-6300, Keyence Co., Osaka, Japan) as shown in Fig. 1. The crack length was given by converting pixels on a crack image on the monitor into millimeters.

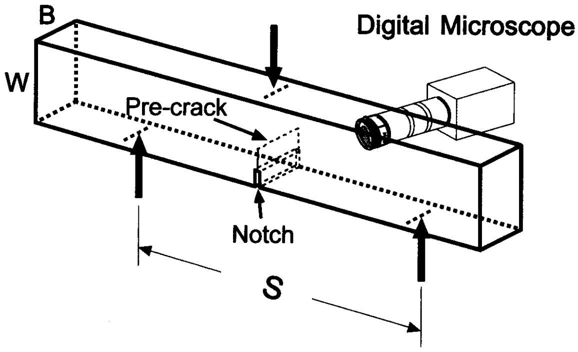

Fig. 1. Illustration of specimen for stable crack growth measurement.
The crack length was obtained from the pixels of a photograph taken from an oblique direction toward the crack surface with an optical digital microscope.

4. Analysis by dimensionless load-load point displacement ($\tilde{P}$-$\tilde{u}$) curve

In an elastic material in which the crack is stably extending, P and u are described by Equations (2) and (3), according to the ASTM E399-78 standard[8] (S = 4W ± 0.2W),

$$\tilde{P}=(K_{IC}\cdot B\cdot W^{1/2})/Y(\alpha)(\alpha=a/W) \quad (2)$$

$$\tilde{u}=C(\alpha)\cdot P=\lambda(\alpha)\cdot P/B\cdot E' \quad (3)$$

where

$$Y(\alpha)=1.5\alpha^{1/2}\{1.99-\alpha(1-\alpha)(2.15-3.93\alpha+2.7\alpha^2)\}/(1+2\alpha)(1-\alpha)^{3/2} \quad (4)$$

and then, $C(\alpha)$ and $\lambda(\alpha)$ are the compliance and the dimensionless compliance as defined by $\lambda(\alpha)=B\cdot E'\cdot C(\alpha)$ with $E'=E/(1-\nu^2)$ for the plain strain respectively, where ν is Poisson's ratio. Sakai et al.[5] introduced the dimensionless load ($\tilde{P}$) and dimensionless load point displacement ($\tilde{u}$) as follows,

$$\tilde{P}=P/K_{IC}\cdot B\cdot W^{1/2} \quad (5)$$

$$\tilde{u}=u\cdot E'/K_{IC}\cdot W^{1/2} \quad (6)$$

using P and u. If the P-u relation is that of an ideally elastic material, Equations (5) and (6) are expressed as follows[5].

$$\tilde{P}=1/Y(\alpha) \quad (7)$$

$$\tilde{u}=\lambda(\alpha)/Y(\alpha) \quad (8)$$

$\lambda(\alpha)$ is calculated from Equation (9)[9~11],

$$\lambda(\alpha)=\lambda(\alpha=0)+2\int Y^2(\alpha)d\alpha \quad (9)$$

with

$$\lambda(\alpha=0)=(S/W)^2[(S/4W)(1+\nu)/2(S/W)] \quad (10)$$

ν is 0.3 to 0.4 in polymer materials. As an influence of fluctuation in ν on Equation (10) is unremarkable, $\nu = 0.3$ was employed in the present analysis.

RESULTS

1. Bending modulus (E)

Fig. 2 indicates an example of the *P-u* curves measured for determining E, where (a) and (b) are for the specimen and the testing system, respectively. The *P-u* relation after taking (b) from (a) was almost linear, and E was yielded as 2.24 ± 0.02 GPa.

2. *P-u* curve in stable crack growth (SCG)

Fig. 3 shows an example of the *P-u* curve that the crack has grown stably at the cross head speed of 0.001 mm/min. The elastic strain in the testing system is also compensated for in this curve. Although this specimen continued crack growth up to the final rupture, many specimens resulted in a rapid failure just after starting the crack growth, or during growth.

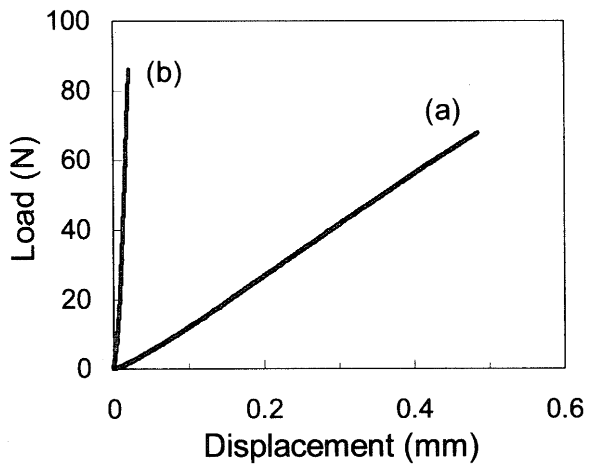

Fig. 2. Load-displacement curves at the cross head speed of 0.001 mm/min.
(a) Specimen in distilled water (b) Apparatus

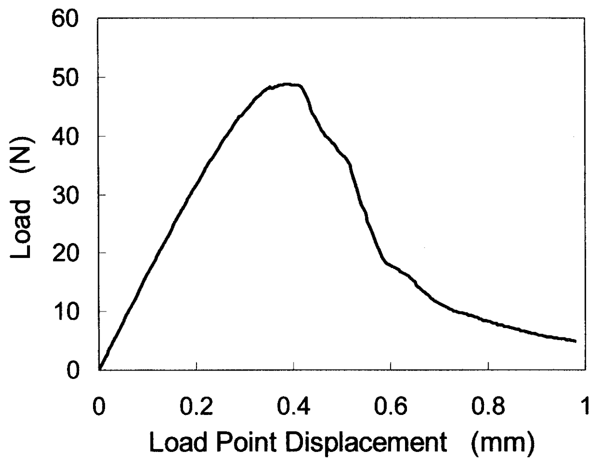

Fig. 3. Load-load point displacement curve in distilled water by the three point bend mode at the cross head speed of 0.001 mm/min.

DISCUSSION

1. Stable crack growth of heat cured denture base resin

The conditions under which the crack can extend stably depends on the compliance of both the specimen ($C_s = u/P$) and the testing system (C_t). The smaller the C_t is, that is, the greater the rigidity of the testing system, the more stably the crack can grow. If C_t is close to C_s, an unstable fracture will take place just after the crack starts to grow. The present C_s (6.92×10^{-6} m/N)/C_t (0.24×10^{-6} m/N) was about 30. Judging from the fact that the rapid failures occurred in many specimens during crack extension, this value is not always satisfactory. The utilization of a highly rigid jig such as a crack arrestor[12] might be useful in order to decrease the elastic strain effect of the testing system.

2. Energetic description of non-linear fracture process

Fig. 4 shows the energy consumed by the non-linear fracture process on the *P-u* diagram in which the unloading and reloading paths are assumed to be straight[13]. If the load is decreased from A, the displacement returns not to zero but to B, and comes back to A again by reloading, getting to D after continued the crack growth. Furthermore, by unloading from D, the displacement moves to C. The total energy ($\Delta\pi_R$) consumed by the crack growth from A to D corresponds to the area, ABCD, and is equal to the resistance energy against the crack growth. The displacement from B to C is irreversible and results from the crack growth during loading from A to D. It is considered that this process accompanies non-linear phenomena of plastic deformation such as drawing out in polymer chains, the formation of the craze, the growth of the microcrack. However, if the crack growth from A to D corresponds to that of a perfect elastic body, the displacement will return to B by unloading from D. Therefore, when point E is

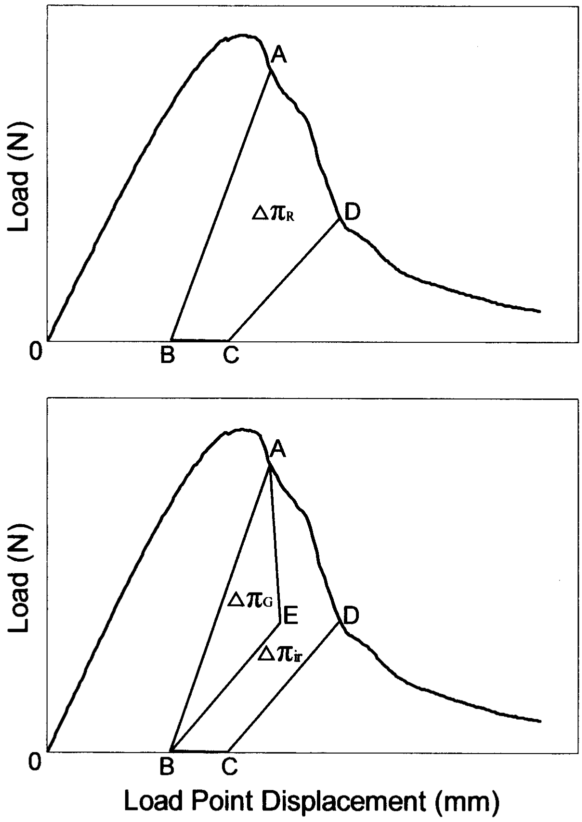

Fig. 4. Non-linear fracture parameters defined by unloading line AB and DC.

made by traveling parallel with the line DC so that point C coincides with point B, the region AEBCD corresponds to the energy ($\Delta\pi_{ir}$) consumed for the formation of irreversible displacement BC. Furthermore, the residual △ABE corresponds to the net energy ($\Delta\pi_G$) required for the formation of the crack surface. These parameters derived from the thermodynamically energy principle[13] for elastic-plastic fractures gives much informations concerning the fracture resistance of a material which shows non-linear fracture behavior. Points B and C can be found directly by the unloading-reloading procedure, but this is undesirable because non-linear fractures such as plastic deformation occur during this procedure. The $\tilde{P}$-$\tilde{u}$ diagram is a method for getting points B and C without unloading.

3. $\tilde{P}$-$\tilde{u}$ curve and non-linear fracture energy parameters

Fig. 5 shows the $\tilde{P}$-$\tilde{u}$ curve calculated from the P-u curve indicated in Fig. 3. The "observed" line was calculated from Equations (5) and (6) using E and $K_{IC} = 1.05 \pm 0.06$ MPa·m$^{1/2}$ measured by Paku et al.[6], and the "theoretical" line on a perfect elastic body was calculated using Equations (7) to (10). In an ideal elastic body, the crack extension starts when the stress intensity factor K_I at the crack tip reaches the critical value K_{IC}. During the crack extension, K_{IC} is kept constant and $\tilde{P}$ can be described by $Y(\alpha)$ only as indicated in Equation (7). Moreover, the relation between $\tilde{P}$ and $\tilde{u}$ is reversible, and $\tilde{u}$ always returns to zero by unloading.

Even if A and D are the arbitrary points on the "observed" line, the locations of both point B and C are determined using the "theoretical" line. The compliances of points A and D are yielded from the slope of 0A′ and 0D′ respectively. Accordingly, points B and C are determined by moving line 0A′ from A′ to A, and line 0D′ from D′ to D, respectively.

Providing that ΔA (<<1) is the crack surface area extended from the point A to D, the non-linear fracture energy parameters (Rc : crack growth re-

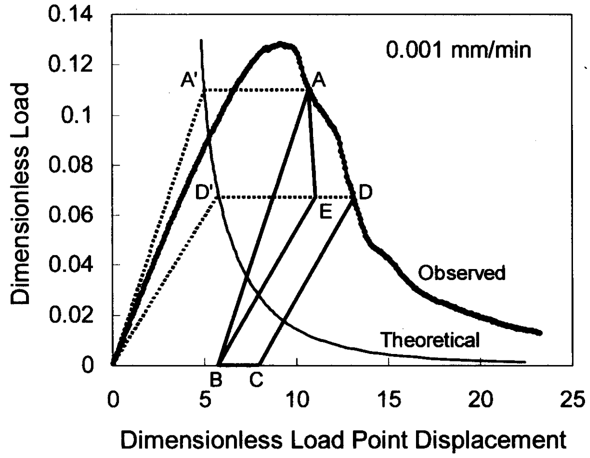

Fig. 5. Dimensionless load-loadpoint displacement curves of heat cured denture base resin (Observed) and the perfect elastic body (Theoretical).

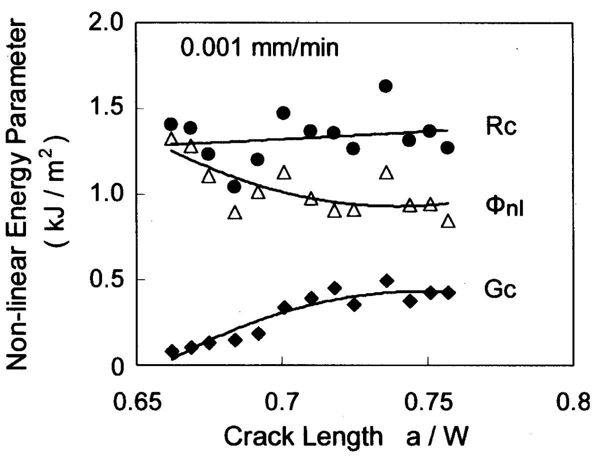

Fig. 6. Non-linear energy fracture parameters of heat cured denture base acrylic resin.
Rc : Crack growth resistance
ϕ_{nl} : Plastic energy dissipation rate
Gc : Non-linear energy fracture toughness

sistance, Gc : non-linear energy fracture toughness and ϕ_{nl} : plastic energy dissipation rate) are yielded by Equations (11),

$$Rc = \triangle \pi_R / \triangle A$$
$$Gc = \triangle \pi_G / \triangle A \quad (11)$$
$$\phi_{nl} = \triangle \pi_{ir} / \triangle A$$

where Rc is the total energy for crack growth, Gc is the net energy necessary for making the crack surface, and ϕ_{nl} is the energy consumed for the plastic deformation in the specimen. These parameters were calculated from the $\tilde{P}$-$\tilde{u}$ curve and the crack length, and shown in Fig. 6. Although Gc and ϕ_{nl} vary with increase of the crack length (a/W), Rc is almost independent. Moreover, the most important result is that ϕ_{nl} accounts for a large part of Rc compared with Gc. The contents of ϕ_{nl} are considered energy dissipated for the formation of craze[14~17] and microcrack. This means that non-linear fracture analysis is essential for investigating fatigue failure or cracking of denture base resin. The present results suggest that the $\tilde{P}$-$\tilde{u}$ curve method can give the quantitative information about the phenomena related to crack growth in dental resin.

CONCLUSION

The application of the dimensionless load-load point displacement ($\tilde{P}$-$\tilde{u}$) curve made it possible to determine the quantitative non-linear fracture energy parameters of heat cured dental base resin. The plastic energy dissipation rate (ϕ_{nl}) was greater than the net energy (Gc) required for the formation of the crack surface, and ϕ_{nl} accounted for a large part of the crack growth resistance (Rc) which is a energy necessary for stable crack growth.

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歯科用レジンの非線形破壊パラメータ解析への無次元荷重ー変位汎曲線の応用

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キーワード：歯科用レジン，非線形破壊パラメータ，無次元荷重－変位汎曲線

抄録　破壊は亀裂先端において塑性変形を伴うものであるから，本質的には非線形現象である．本研究では歯科用レジンの非線形破壊の解析方法を確立する目的で，義歯床用加熱重合レジンに無次元荷重－変位汎曲線（$\tilde{P}$-$\tilde{u}$曲線）の適用を試みた．その結果，非線形破壊エネルギーパラメータ（亀裂成長抵抗値Rc，非線形エネルギー破壊靱性値Gc，塑性エネルギー散逸値ϕ_{nl}）の定量化が可能なことが明らかになり，義歯床用加熱重合レジンの場合，ϕ_{nl}は亀裂面を生成するために必要な正味のエネルギーGcより大きく，亀裂成長に必要な全エネルギーRcの大きな部分を占めることが分かった．このことから$\tilde{P}$-$\tilde{u}$曲線は歯科用レジンの疲労破壊のエネルギー的解析を可能にし，床の割れの原因の解明と対策に有用であると考えられる．